Appendix F: General Algebraic Representation of the Biogenic Assessment Factor Equations

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1. Introduction

The equations and conceptual diagrams in this appendix are intended to illustrate the various carbon flows that contribute to the calculation of net atmospheric contribution of biogenic CO_2 emissions associated with the production, processing, and use of biogenic material at a stationary source *(NBE)* and biogenic assessment factor (*BAF*) values for stationary source biogenic feedstock consumption. This appendix builds on the primary *NBE* equation provided in the main report to develop a generic algebraic formulation describing how the net atmospheric contribution of biogenic CO_2 emissions for a stationary source could be calculated at different points of assessment, depending on the specific policy or programmatic context, and to provide simple concrete examples of how the generic equation could be applied.

2. Simple Algebraic Representation

As discussed throughout this report, the Net Biogenic Emissions *(NBE)* from stationary source biomass consumption equation can be presented as the following equation:

$$\boldsymbol{NBE} = (\boldsymbol{PGE})(\boldsymbol{GROW} + \boldsymbol{AVOIDEMIT} + \boldsymbol{SITETNC} + \boldsymbol{LEAK})(\boldsymbol{L})(\boldsymbol{P}) = (\boldsymbol{PGE})(\boldsymbol{BAF}) \quad \textbf{(EQ. F.1)}$$

Where the *BAF* is given by Equation F.2:

$$\boldsymbol{BAF = \frac{NBE}{PGE} = (GROW + AVOIDEMIT + SITETNC + LEAK)(L)(P)} \qquad \textbf{(EQ. F.2)}$$

The equations above are designed to transform a measurable or estimated quantity (the carbon content of the biogenic feedstock used at the point of assessment [potential gross emissions or *PGE*]) into a quantity that cannot be directly measured (the net atmospheric biogenic CO_2 contributions associated with different stages of biogenic feedstock production, processing, and use at a stationary source [*NBE*]). The terms in the *NBE* equation each play a specific role in this transformation:

- *PGE* is the carbon content of the biogenic feedstock used by a specific entity or generally consumed). This is a quantity that could be measured or estimated at different points of assessment (e.g., at the boiler mouth, stationary source gate, feedstock production site, or at the stack: wherever the point of assessment needs to be. Thus, this term can have different values indicated by subscripts, representing different points along the supply chain).
- *L* is a unitless adjustment factor greater than or equal to one that represents biogenic feedstock carbon that leaves the supply chain (e.g., via transit or decomposition, deviated for use as a product) between the feedstock production site and input into the conversion process at a stationary source. *L* scales *PGE*, as it was measured at the point of assessment, up to account for any losses during transportation or storage between the feedstock production site and the point of assessment. *PGE* times *L* is thus the carbon content of the biomass that was grown at the feedstock production site in order to deliver the quantity of feedstock measured at the point of assessment.
- *P* is a unitless adjustment factor between zero and one, equal to the share of the carbon content of the feedstock at the point of assessment that is emitted to the atmosphere by a stationary source (versus that which is embedded in products). In effect, this term also reflects the share of carbon that remains in products, that is either not emitted to the atmosphere or is sold and eventually emitted to the atmosphere by a downstream user.
- *(GROW + AVOIDEMIT + SITETNC + LEAK)* represents the landscape emissions effect. This landscape emissions effect is the sum of four unitless factors that relate the total biogenic carbon content of the feedstock grown at the feedstock production site, i.e. *(PGE)*(L)*, to related landscape biogenic carbon pools. The details of these terms are discussed elsewhere in this framework. For the purposes of this appendix, we can think of the terms *(GROW + AVOIDEMIT + SITETNC + LEAK)*(PGE)*(L)* as the estimated net contribution to the atmosphere associated with growing, harvesting, producing, processing, and using the feedstock that was measured at the point of assessment. This amount is then multiplied by *P* to determine the share that is actually emitted and is the responsibility of a particular entity.

The framework itself is designed to be flexible enough such that it can be applied to a variety of programs with different requirements. For example, *PGE* in the *NBE* equation above is the potential gross emissions at the point of assessment for the purposes of applying the biogenic assessment factor. Depending on how the framework is applied, this point of assessment could be interpreted as biogenic emissions at the boiler mouth, the stack, or total potential biogenic emissions at the farm gate once biomass has been harvested. Additionally, losses can occur and products can be produced at different points along the supply chain.

To understand how the *NBE* equation adapts to different points of assessment and accounts for products and losses at different points along the supply chain, we need to understand how the *L* and *P* terms are calculated and what exactly they represent. In order to build this understanding, this appendix begins with a simple example and then shows how the generic version of the equation is able to capture more complexity in subsequent examples.

2.1. Example 1: Simple Carbon Flow with Point of Assessment at the Boiler/Fermenter Mouth

Consider the following conceptual example provided by Figure F-1. This flow diagram shows the evolution of *PGE* along the production supply chain. This representation uses atmospheric accounting methods, so terms that represent an emission (or potential emission) to the atmosphere (e.g., *LOSS, PGE*) are positive, and terms that represent sequestration (e.g., *PROD*) are negative.

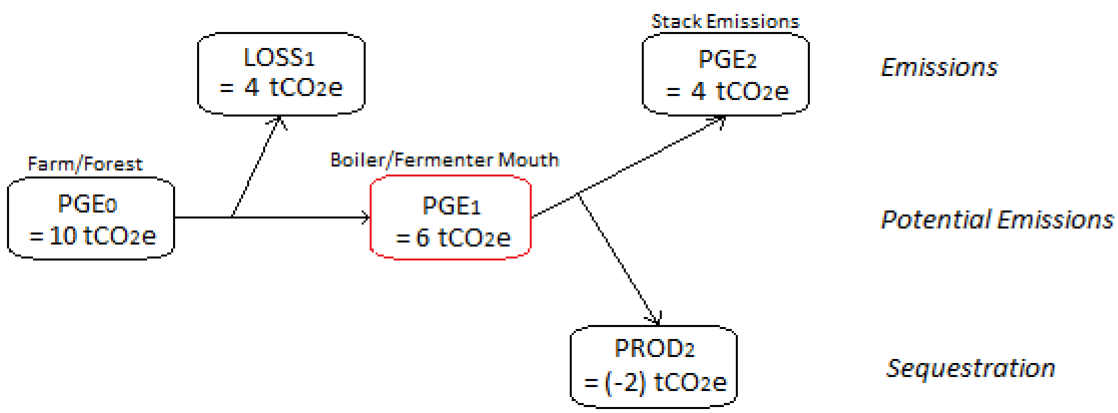

Figure F-1: Conceptual Diagram Illustrating the Calculation of Potential Gross Emissions at Different Points of Assessment.

The first thing to note in this diagram is that it contains different variables from the variables in Equations F.1 and F.2 such as *LOSS* and *PROD*. The main equations discussed above are designed to be applied with limited data requirements. The example here is designed as a thought experiment to "follow the tons of biogenic CO_2," and we show how these quantities can be used to calculate the variables that go into the *BAF* equation. Another difference is that the *PGE* variables gain subscripts to represent the different points of assessment at which *PGE* could be measured (e.g., at the forest/farm, at the boiler mouth, out the stack). This can be generalized to PGE_i in order cover any number of measurement points, where *i* indexes over all the points of measurement. The red circle indicates where the point of assessment is in this example, so here PGE_1 would be equal to *PGE* from Equations F.1 and F.2 as applied at the boiler/fermenter mouth.

The terms *LOSS* and *PROD* represent the actual tons of carbon lost in transportation or storage along the supply chain (*LOSS*) or stored in final products or by-products (*PROD*) (rather than emitted during conversion). The subscripts indicate at which stage in the supply chain each occurs, e.g., $LOSS_1$ occurs before PGE_1, and $PROD_2$ occurs before PGE_2. These subscripts will become necessary when we move to more complex supply chain examples and generalize the equations. In the schematic of this example (Figure F.1), the top row represents actual gross emissions to the atmosphere. Here losses are assumed to generate actual emissions, and PGE_2 represents actual emissions out the stack (note that at this point the emissions are no longer "potential," but for notational reasons we still refer to these stack emissions as *PGE*). The middle row is for potential emissions at various assessment points. These values represent tons of biomass moving through the supply chain that have not yet been emitted or sequestered or contained in products. The bottom line of Figure F.1 is tons that go into products or by-products and are either not emitted to the atmosphere or not emitted by this entity. Note that because we are using atmospheric accounting, emissions and potential gross emissions have a positive value, and carbon contained in products or by-products is assigned a negative value.

- PGE_0: Represents *PGE* at the forest/farm gate. This is the total harvested biogenic CO_2 with the potential to be emitted at the forest/farm gate and includes all biomass that is harvested and transported from the forest or farm to the stationary source facility.
- $LOSS_1$: Represents the biogenic CO_2 lost in transportation or storage between the forest/farm gate (PGE_0) and the boiler/fermenter mouth (PGE_1).
- PGE_1: Represents *PGE* at the boiler/fermenter mouth.
- $PROD_2$: Represents biogenic CO_2 stored in long-term product pools (including lumber, ethanol, or other purely marketable products produced with a portion of the harvested biomass) or other nonmarketable industrial by-products (including ash).
- PGE_2: Represents emissions at the stack.

Thus, the equations relating these variables are:

$$PGE_1 = PGE_0 - LOSS_1 = 10 - 4 = 6 \text{ tCO}_2\text{e.} \quad \textbf{(EQ. F.3)}$$

$$PGE_2 = PGE_1 + PROD_2 = 6 + (-2) = 4 \text{ tCO}_2\text{e.} \quad \textbf{(EQ. F.4)}$$

Now let us define how the terms in the *NBE* equation *(GROW + AVOIDEMIT + SITETNC + LEAK), (L),* and *(P)* are calculated and apply our example values. Note that in the equations below we adopt the notation convention that *PGE* at the point of assessment (PGE_1 in this example, or simply *PGE* in Equations F.1 and F.2) is written as PGE_j, where the subscript "*j*" represents the point of assessment (in this example, j=1, the boiler/fermenter mouth).

The landscape emissions effects terms (*GROW, AVOIDEMIT, SITETNC, and LEAK*) are discussed extensively elsewhere in the framework, including detailed discussions of how they can be calculated in practice. It is important to note that *GROW, AVOIDEMIT, SITETNC*, and *LEAK* are all unitless.[1] For the purposes of this appendix, we can define our landscape emissions effects terms from the overall *BAF* equation as follows:

[1] However, the framework can be adapted to use units instead of unitless values as needed for a specific application.

$$(\boldsymbol{GROW} + \boldsymbol{AVOIDEMIT} + \boldsymbol{SITETNC} + \boldsymbol{LEAK}) = \left(\frac{\boldsymbol{G+A+S+Lk}}{\boldsymbol{PGE_0}}\right). \qquad \textbf{(EQ. F.5)}$$

This is merely another way to specify the landscape effect. The original term is a unitless ratio of the contribution of landscape effects on the overall *BAF* value. However, we can also think of each of these elements as being relative to the total amount of biomass that is harvested at the forest or farm (PGE_0). *G, A, S,* and *Lk* are variables that represent the actual tons of landscape net emissions resulting from producing PGE_0 tons of biomass.[2] In that sense, *(G+A+S+Lk)* represents actual net emissions on the landscape caused by a harvest of PGE_0. The landscape-level emissions are normalized by PGE_0 to arrive at the original, unitless term.

Now let's consider the terms *L* and *P* from the *NBE* equations. *L* accounts for transportation or storage losses, relating the carbon content of biomass used by the facility at the boiler fermenter mouth (PGE_1) to the carbon content of that biomass when it was grown and harvested at the forest/farm (PGE_0).

L is defined as the ratio of harvested potential biogenic emissions (PGE_0) and *PGE* evaluated at assessment point *j*.

$$\boldsymbol{L} = \left(\frac{\boldsymbol{PGE_0}}{\boldsymbol{PGE_j}}\right) \qquad \textbf{(EQ. F.6)}$$

Note that this fraction will result in a positive number greater than or equal to one. Also note that although *LOSS* does not appear in this equation, we could substitute in from equation 0 to express *L* in terms of just one of the *PGE* variables and the *LOSS* term.

The purpose of *L* here is twofold. First, it represents the transformation of PGE_j to PGE_0, implicitly capturing any losses and products[3] that occur between the point of assessment and the forest/farm. Furthermore, if considered in combination with the landscape effects term, it serves to bring the landscape effects in relation to PGE_j. That is, while landscape-level terms implicitly capture the emissions impact of total biogenic feedstock harvests (PGE_0), *L* provides the necessary adjustment for total potential biogenic emissions within the stationary source boundary.

P accounts for biogenic feedstock carbon embodied in process products that pass out of the supply chain as product prior to combustion or after combustion by exiting the stationary source through forms other than as stack emissions. *P* is a unitless term between zero and one that scales *PGE* down so that the portion of biogenic feedstock carbon embodied in products or byproducts is not included in the final results when calculating the assessment factor for biogenic CO_2 emissions. As shown in the subsequent sections of this appendix, the equation used to calculate *P* can become quite complex depending on the point of assessment and the production supply chain. This example

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[2] Where $GROW = G / PGE_0$; $AVOIDEMIT = A / PGE_0$; $SITETNC = S / PGE_0$; and $LEAK = Lk / PGE_0$.

[3] In the example presented here in section 1.1 only losses occur between the point of assessment and the forest/farm, so *L* only needs to account for these losses. However, if any products or by-products are produced between the forest/farm and the point of assessment (as in the example presented later in section 1.5), then in order to scale PGE_i up to PGE_0, *L* needs to account for these products, as well as any losses. The responsibility for the carbon embodied in these products will separately be accounted for in the *P* term as discussed below.

is constructed to produce the most simplified version of the *P* equation. Equation F.7 defines *P* as calculated in this simplified example:

$$\boldsymbol{P = 1 + \frac{\sum_{i=1}^{S} PROD_i}{PGE_j}}. \qquad \textbf{(EQ. F.7)}$$

Note that for many stationary sources, biomass will be used to produce a number of final products, so the summation in equation 0 simply sums over all *i* products that are produced, indexed from 1 to *S*, where *S* is the last point on the carbon trail and represents the stack. In this example, $PROD_2$ is the only product, so this summation would simply be equal to $PROD_2$. Because $PROD_i$ is always negative and the absolute value of the sum of all $PROD_i$ must be less than PGE_j, the sum of all $PROD_i$ divided by PGE_j will be a fraction between negative one and zero, and *P* will be a positive number between zero and one.

P is one minus the share of PGE_j that is sequestered in products or by-products and is either not emitted to the atmosphere or is sold and eventually emitted to the atmosphere by a different entity. (Note that because *PROD* is negative, technically this expression is one plus a negative number.) In effect, we are taking away the portion of carbon the products or by-products are responsible for and attributing the remainder to the facility. Although this is a relatively simple example, the mathematical representation of this term will change as we consider more complicated scenarios.

We can now rewrite the equation for *NBE* by substituting into Equation F.1. from Equations F.5, F.6, and F.7, resulting in the following:

$$\boldsymbol{NBE = PGE_j \left(\frac{G+A+S+Lk}{PGE_0}\right)\left(\frac{PGE_0}{PGE_j}\right)\left(1 + \frac{\sum_{i=1}^{S} PROD_i}{PGE_j}\right)}. \qquad \textbf{(EQ. F.8)}$$

The PGE_0 in numerator of the *L* term cancels with the PGE_0 in the ratio of $(G+A+S+Lk)/\ PGE_0$, and the PGE_j in the denominator of the *L* term cancels with the first PGE_j in the *NBE* equation, so that the term *(GROW + AVOIDEMIT + SITETNC + LEAK)* (PGE)*(L) = G+A+S+Lk*. As discussed above, this represents the actual net emissions on the landscape caused by a harvest of PGE_0. All of this canceling is important, because *G, A, S,* and *Lk* are not observable, and even PGE_0 may not be readily observable, but we can estimate the unitless values of *L, GROW, AVOIDEMIT, SITETNC,* and *LEAK* through other means (such as retrospective reference point or future anticipated baseline modeling methods) and calculate net emissions while only needing to observe PGE_j.

Now, applying numerical values from Figure F-1 and assuming that (*G+A+S+Lk*) = 3 tCO_2e (for the purpose of these examples), net biogenic emissions at point of assessment "*j*=1" are calculated as:

$$\boldsymbol{NBE = 6\left(\frac{3}{10}\right)\left(\frac{10}{6}\right)\left(1 + \frac{(-2)}{6}\right) = 3\left(\frac{4}{6}\right) = 2\ tCO_2e}. \qquad \textbf{(EQ. F.9)}$$

2.2. Example 2: Changing the Point of Assessment to Stack Emissions

Continuing through the following examples, necessary modifications are made to the BAF equation for alternative points of assessment. The biogenic CO_2 trail depicted by Figure F-2 is used in the following examples to demonstrate that the same NBE can be calculated regardless of the point of assessment. This provides a consistency check because this exercise merely alters the point of assessment but not the biogenic carbon trail.

The second example also relies on the simple hypothetical carbon trail from Figure F-1, with the only difference being that the new point of assessment is stack emissions (represented by PGE_2), i.e., "j=2."

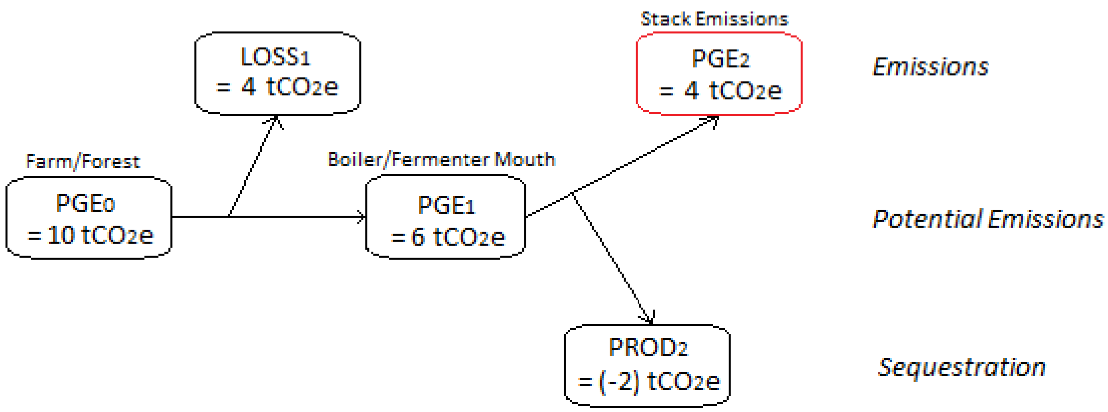

Figure F-2: Stack Emissions as the Point of Assessment.

In the first example, all losses occurred before the point of assessment, and all products were produced after the point of assessment. In this example, both losses and products occur before the point of assessment. This change will require us to refine our interpretation of *L* and rewrite our definition of *P* in more general form, as shown in Equation F.10. Equation F.11 is the new more generalized *NBE* equation that covers this example and incorporates this revised definition of *P*. Note that example one is a special case of this new equation.

$$\boldsymbol{P = 1 + \frac{\sum_{i=1}^{S} PROD_i}{PGE_j - \sum_{i=1}^{j} PROD_i}} \quad \textbf{(EQ. F.10)}$$

$$\boldsymbol{NBE = PGE_j \left(\frac{G+A+S+Lk}{PGE_0}\right)\left(\frac{PGE_0}{PGE_j}\right)\left[1 + \frac{\sum_{i=1}^{S} PROD_i}{PGE_j - \sum_{i=1}^{j} PROD_i}\right]} \quad \textbf{(EQ. F.11)}$$

The first point to notice is that *L* remains the ratio of PGE_0 to PGE_j. Although the expression for *L* has not changed, the numerical value and the interpretation will be different, because PGE_j is now stack emissions. In our original interpretation, *L* was a unitless adjustment factor greater than or equal to one, that scales *PGE* as it was measured at the point of assessment up to account for any losses during transportation or storage between the forest/farm and the point of assessment. In a more general representation, when there can be both products and losses before the point of assessment, *L* scales *PGE* as it was measured at the point of assessment up to account for any losses during transportation or storage between the forest/farm and the point of assessment and to account for any products produced between the forest/farm and the point of assessment. In the general form, *L* does more than just account for losses, it serves as a general scaling factor to relate the carbon content of the biomass feedstock at the point of assessment (PGE_j) to the carbon content of biomass that was grown and harvested at the forest/farm and that was required to generate that feedstock (PGE_0), accounting for all differences between those two points of measurement.

As in the previous example, *(GROW + AVOIDEMIT + SITETNC + LEAK)* (PGE)*(L) = G+A+S+Lk* represents the primary portion of the equation, the net emissions on the landscape caused by a harvest of PGE_0. The remaining job for the *P* term is to determine what share of these net landscape emissions a particular facility is responsible for. The *P* definition from Equation F.7 must be altered to ensure that products that are processed *before* the point of assessment are properly accounted for, resulting in the new definition of *P* in Equation F.10. To do so, the denominator in the fraction used to calculate *P* must be adjusted. Instead of representing the ratio of all products produced to PGE_j, this new fraction deducts contributions from products that occur prior to the point of assessment from PGE_j in the denominator $(PGE_j - \sum_{i=1}^{j} PROD_i)$. Thinking back to Equation F.4, $PGE_j = PGE_{j-1} + PROD_j$, rearranging this expression and generalizing, the new denominator is simply the expression for PGE_{j-1}, so even though the point of assessment is now at the stack, after the products have been produced, our expression for *P* still needs to consider products as a share of *PGE* at a point before the products were produced.

The following numerical example illustrates this point. Because all that has changed in this example is the point of assessment, the value calculated for *NBE* should be the same as in the first example. Substituting numerical values from Figure F-1, and again assuming that (*G+A+S+Lk*) = 3 tCO_2e, *NBE* is calculated in equation 0.

$$\boldsymbol{NBE = 4\left(\frac{3}{10}\right)\left(\frac{10}{4}\right)\left[1+\frac{(-2)}{4-(-2)}\right] = 3\left[1-\frac{2}{6}\right] = 3\left(\frac{4}{6}\right) = 2\ mtCO_2e} \qquad \textbf{(EQ. F.12)}$$

Note that although the point of assessment has changed, net biogenic emissions remain consistent with the previous example that evaluated *PGE* at the boiler/fermenter mouth.

2.3. Example 3: Changing the Point of Assessment to Forest/Farm Gate

The third example (Figure F-3) also relies on the simple hypothetical carbon trail from Figures F-1 and F-2, with the only difference being that the new point of assessment is the forest/farm (represented by PGE_0), i.e., *j*=0.

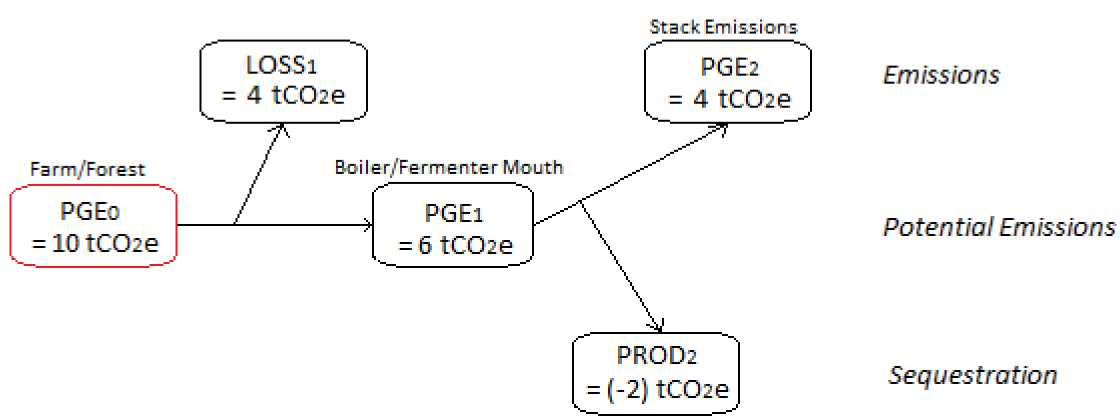

Figure F-3: Forest/Farm as the Point of Assessment.

In this example, because the point of assessment is at PGE_0, the *L* term is equal to one. The *L* term is still theoretically scaling between the point of assessment and the forest/farm level, but as they are the same, the L term has a value of one. PGE_0 is directly measured as it is PGE_j in this instance, so PGE_j does not need to be scaled up, and L is simply equal to one.

In this example, losses ($LOSS_1$) do not enter into the L term, but because the mass balance has not changed, losses still are occurring, but they are occurring after the point of assessment, so the question is how are losses accounted for now? In all examples, (PGE_j)*(L) = PGE_0. The equation then assumes that all of PGE_0 is emitted, unless the P term specifies that a share of PGE_0 is sequestered or contained in products. This means that once PGE_j is transformed to PGE_0, unless there are products, the calculation does not change regardless of whether the emissions come from a loss or from the stack. Consider for a moment an example without products (i.e., P=1); here $NBE = PGE_j\left(\frac{G+A+S+Lk}{PGE_0}\right)\left(\frac{PGE_0}{PGE_j}\right)$, and whether the point of assessment is before or after the losses occur (i.e., j = 0 or 1; or, in this example, at the forest/farm or at the boiler/fermenter mouth), either way, all the PGE terms cancel so that $NBE = G+A+S+Lk$.

Returning to the example shown in figure 3, although losses do not directly enter the L term in this example, they do enter the P term. In this instance, the equation must account for losses that occur after the point of assessment j, but before the first source of products, in the P term (as $LOSS_1$ does in this example). Similar to the previous example, a more generic form of the P term is applied. The calculation of P is modified such that the denominator accounts for the losses that occur after the point of assessment, but before the first product is produced. Similar to the previous example, in the denominator of the fraction in the P term, losses are deducted from PGE_j in the denominator ($PGE_j - \sum_{i=j+1}^{PR} LOSS_i$), and as demonstrated in Equation 0F.13, this is essentially the expression for PGE_{j+1}. In this case, the term PR stands for the process stage i where the first product is made (that is, the first stage of the industrial process in which $PROD_i > 0$).

In effect, as the P term ensures that the facility is not held responsible for the portion of biogenic carbon that is passed on embodied in products or by-products, the P term also ensures that the facility is not held responsible for the portion of the losses that the products or by-products are responsible for. The responsibility for the losses is shared between the facility and the products or by-products in proportion to their respective shares or biomass that remains after the losses occurred. Equation F.14 presents the NBE equation for this scenario.

$$\boldsymbol{P = 1 + \frac{\sum_{i=1}^{S} PROD_i}{PGE_j - \sum_{i=j+1}^{PR} LOSS_i}} \qquad \textbf{(EQ. F.13)}$$

$$\boldsymbol{NBE = PGE_j\left(\frac{G+A+S+Lk}{PGE_0}\right)\left(\frac{PGE_0}{PGE_j}\right)\left[1 + \frac{\sum_{i=1}^{S} PROD_i}{PGE_j - \sum_{i=j+1}^{PR} LOSS_i}\right]} \qquad \textbf{(EQ. F.14)}$$

Substituting numerical values from Figure F-1, and again assuming that *(G+A+S+Lk)* = 3 tCO_2e, yields the following NBE calculation, which generates the same value for NBE using the more general equation in this example:

$$\boldsymbol{NBE = 10\left(\frac{3}{10}\right)\left(\frac{10}{10}\right)\left[1 + \frac{(-2)}{10-4}\right] = 3\left[1 - \frac{2}{6}\right] = 3\left(\frac{4}{6}\right) = 2\ tCO_2e} \qquad \textbf{(EQ. F.15)}$$

2.4. Example 4: Combining Forms from Examples 2 and 3

In order to write a more general form of the *NBE* equation that can either account for products that occur *before* the point of assessment or any losses generated before any products are produced, both forms of the equation as established in examples 2 and 3 must be combined. Doing so results in Equation 0F.17. This generalized form can be applied to all the previous examples of biogenic CO_2 calculation at different points of assessment. Note that S in the equations below represents the last point on the carbon trail; i.e. the stack, so PGE_S would be stack emissions.

$$\boldsymbol{P = 1 + \frac{\sum_{i=1}^{S} PROD_i}{PGE_j - \sum_{i=1}^{j} PROD_i - \sum_{i=j}^{PR} LOSS_i}} \qquad \textbf{(EQ. F.16)}$$

$$\boldsymbol{NBE = PGE_j \left(\frac{G+A+S+Lk}{PGE_0}\right)\left(\frac{PGE_0}{PGE_j}\right)\left[1 + \frac{\sum_{i=1}^{S} PROD_i}{PGE_j - \sum_{i=1}^{j} PROD_i - \sum_{i=j}^{PR} LOSS_i}\right].} \qquad \textbf{(EQ. F.17)}$$

Equation F.17 is general enough to cover all points of assessment for any carbon chain so long as all losses ($LOSS_i$) occur before the first product ($PROD_i$) is generated.

As explained in the previous examples, the denominator in the fraction in the P term uses the relationships in Equations F.3 and F.4 to transform PGE_j into PGE at another point in the carbon trail. Before providing a more complex carbon trail scenario in example 5, it will be useful to introduce some new subscript notation and be a bit more explicit about the subscript notation used so far.

- 0: First point on the carbon trail; represents the forest/farm. *PGE* begins at 0.
- S: Last point on the carbon trail; represents the stack. PGE_S is stack emissions.
- PR: Index number for the first product produced on the carbon trail. $PROD_{PR}$ is the first product.

The indexing for points along the carbon trail begins at 0, the forest/farm, and ends at S, the stack. The convention for indexing *LOSS* and *PROD* is that they take an index number equal to the next *PGE*, and a *LOSS* and a *PROD* cannot both occur at the same stage.[4] Additionally, any loss that occurs after all products are produced is indistinguishable from stack emissions PGE_s in these equations. To simplify the equation, it is assumed that *LOSS* is not specified after the last product is produced in the carbon trail, and instead any such losses are rolled into the calculation of PGE_s.

With this notation in mind, Equations F.16 and F.17 are rewritten as:

$$\boldsymbol{P = 1 + \frac{PROD_{Total}}{PGE_{PR-1}}} \qquad \textbf{(EQ. F.18)}$$

[4] If $LOSS_i$ and $PROD_i$ occurred at the same stage i, it would be ambiguous which came first and whether that $PROD_i$ should share responsibility for $LOSS_i$ or not. For purposes of specifying the full theoretical carbon trail, a PGE would need to be inserted between $LOSS_i$ and a $PROD_i$.

$$NBE = PGE_j \left(\frac{G+A+S+Lk}{PGE_0}\right)\left(\frac{PGE_0}{PGE_j}\right)\left[1+\frac{PROD_{Total}}{PGE_{PR-1}}\right], \quad \textbf{(EQ. F.19)}$$

Where we also define a new variable, $PROD_{Total} = \sum_{i=1}^{S} PROD_i$, or the sum of all products produced along the carbon supply chain. PGE_{PR-1} is potential gross emissions at the point in the carbon supply chain before the last product is produced. In examples 1 through 3 above, PGE_{PR-1} = PGE_1. With this new notation, the expression for P in Equation F.18 does not vary with different points of assessment.

The examples so far all assume that losses occur before any products are produced in the carbon supply chain. If a product is produced at a point in the carbon chain before a loss occurs, then that product should not be held responsible for the subsequent loss. Example 5 below shows how the *NBE* equation can be fully generalized to cover this situation.

2.5. Example 5: Extending to a More Complex Biogenic CO_2 Trail—Toward Fully Generalizing the *NBE* Equation

To illustrate how one could apply the *NBE* equation to a more complex carbon trail, consider the more complex hypothetical carbon trail in Figure F-4. The primary differences between this conceptual diagram and the previous examples are that this example includes four potential points of assessment for PGE_j, and multiple points where products are produced and losses occur.

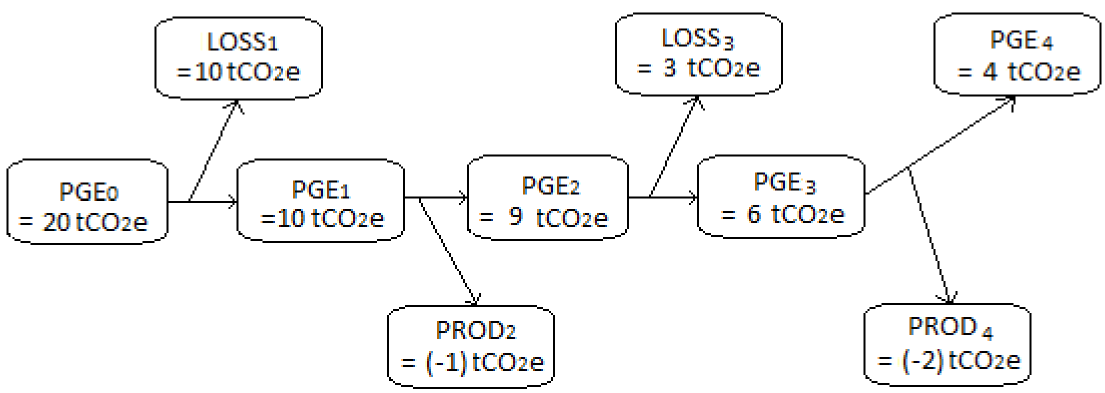

Figure F-4: Conceptual Diagram Illustrating a More Complex Carbon Trail.

If one attempts to calculate the P term in the *NBE* equation for this example using Equation F.180, the resulting value would be $P = 1 + PROD_{Total} / PGE_1 = 1+(-3/10) = 7/10$. This would be incorrect though, because it assigns the facility full responsibility for the $LOSS_3$ term and thus does not account for the portion of $LOSS_3$ that $PROD_4$ is accountable for. To properly account for how responsibility for $LOSS_3$ is shared, one should apply the following equations for P and *NBE*:

$$P = 1 + \frac{PROD_{Total} + \sum_{i=PR}^{S}\left[LOSS_i\left(\frac{\sum_{k=i}^{S} PROD_k}{PGE_i}\right)\right]}{PGE_{PR-1}} \quad \textbf{(EQ. F.20)}$$

$$NBE = PGE_j \left(\frac{G+A+S+Lk}{PGE_0}\right)\left(\frac{PGE_0}{PGE_j}\right)\left[1+\frac{PROD_{Total} + \sum_{i=PR}^{S}\left[LOSS_i\left(\frac{\sum_{k=i}^{S} PROD_k}{PGE_i}\right)\right]}{PGE_{PR-1}}\right] \quad \textbf{(EQ. F.21)}$$

Assuming *j*=1, adding numerical values, and again assuming that *(G+A+S+Lk)* = 3 tCO_2e, gives the following calculation of NBE:

$$\boldsymbol{NBE = 10\left(\frac{3}{20}\right)\left(\frac{20}{10}\right)\left[1+\frac{(-3)+(3)\left(\frac{-2}{6}\right)}{10}\right] = 3\left[1-\frac{3+3\left(\frac{1}{3}\right)}{10}\right] = 3\left[1-\frac{4}{10}\right] = 3\left[\frac{6}{10}\right] = 1.8} \quad \textbf{(EQ. F.22)}$$

This example is more complicated than the earlier examples but can still be thought of intuitively. At the end of the carbon trail, PGE_4 and $PROD_4$ are responsible for 4 tCO_2e and 2 tCO_2e of PGE_3, respectively. They share responsibility for $LOSS_3$ proportionally, 4/6 for PGE_4 and 2/6 for $PROD_4$. So PGE_4 is responsible for 2 tCO_2e of $LOSS_3$ and $PROD_4$ is responsible for 1 tCO_2e of $LOSS_3$. Put these together and PGE_4 is responsible for 6 tCO_2e of PGE_2 and $PROD_4$ is responsible for 3 tCO_2e of PGE_2. These are also the quantities they are responsible for of PGE_1, because the only difference between PGE_1 and PGE_2 is the 1 tCO_2e that $PROD_2$ is responsible for. So PGE_4 is responsible for 6/10 of PGE_1. As shown in the calculation above in Equation F.22, this confirms that *P* = 6/10. Note that because $LOSS_1$ occurs before all products, responsibility for it is shared in the same proportion as responsibility for PGE_{PR-1}, so it does not need to be accounted for in *P*.[5]

Although Equation F.21 is a more general representation of NBE than the previous examples, it does not cover all possible scenarios. If the carbon chain is more complex, and there are multiple losses after the first product, then the $\sum_{k=i}^{S} PROD_k$ term in the numerator of the fraction that shares out responsibility for $LOSS_i$ will not account for $PROD_k$'s share of any subsequent losses, instead assigning full responsibility for those subsequent losses to PGE_s. Instead of demonstrating this with another example, the same effect can be illustrated by calculating *P* with respect to PGE_0 instead of PGE_{PR-1}. This is not a more general way of calculating *P,* but simply an equivalent expression. With this change, one needs to account for $LOSS_1$ in *P*, resulting in the following equations:

$$\boldsymbol{P = 1 + \frac{PROD_{Total}+\sum_{i=1}^{S}\left[LOSS_i\left(\frac{\sum_{k=i}^{S}\left(PROD_k+\sum_{l=i+1}^{k}\left(LOSS_l\frac{PROD_k}{PGE_l}\right)\right)}{PGE_i}\right)\right]}{PGE_0}} \quad \textbf{(EQ. F.23)}$$

$$\boldsymbol{NBE = PGE_j\left(\frac{G+A+S+Lk}{PGE_0}\right)\left(\frac{PGE_0}{PGE_j}\right)\left[1+\frac{PROD_{Total}+\sum_{i=1}^{S}\left[LOSS_i\left(\frac{\sum_{k=i}^{S}\left(PROD_k+\sum_{l=i+1}^{k}\left(LOSS_l\frac{PROD_k}{PGE_l}\right)\right)}{PGE_i}\right)\right]}{PGE_0}\right]}. \quad \textbf{(EQ. F.24)}$$

Again assuming *j*=1 and adding numerical values gives the following calculation of NBE:

$$\boldsymbol{NBE = 10\left(\frac{3}{20}\right)\left(\frac{20}{10}\right)\left[1+\frac{(-3)+(10)\left(\frac{(-1)+\left(-2+3\frac{(-2)}{6}\right)}{10}\right)+(3)\left(\frac{-2}{6}\right)}{20}\right]} \quad \textbf{(EQ. F.25)}$$

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[5] Note that while $LOSS_1$ does not show up in the *P* term in this example, it does not drop out of the *NBE* calculation entirely as it is accounted for in the *L* term.

$$\boldsymbol{NBE} = \mathbf{3}\left[\mathbf{1} - \frac{\mathbf{3+10\left(\frac{1+3}{10}\right)+3\left(\frac{1}{3}\right)}}{\mathbf{20}}\right] = \mathbf{3}\left[\mathbf{1} - \frac{\mathbf{3+4+1}}{\mathbf{20}}\right] = \mathbf{3}\left[\mathbf{1} - \frac{\mathbf{8}}{\mathbf{20}}\right] = \mathbf{3}\left[\frac{\mathbf{12}}{\mathbf{20}}\right] = \mathbf{1.8}. \qquad \textbf{(EQ. F.26)}$$

As shown in this numerical example, this expanded equation still generates the same answer for *NBE* in this example.

More complex supply chain carbon trails will require continually more complex expressions for *P.* Comparing the two equivalent expressions above for *P* in Equations F.20 and F.23, defining *P* in relation to $PGE_{PR\text{-}1}$ required an additional nested summation. Adding losses and products to the example will similarly require additional nested summations to fully assign responsibility for subsequent losses to the products and by-products in the supply chain. An important note about the additional complexity required to fully account for a more complex supply chain, is that using the more complex version of the equation (e.g., Equation F.21 instead of Equation F.19) will always assign more responsibility for the losses to products and by-products, thus lowering the calculated value of *P*, lowering the calculated *NBE*, and lowering the ultimate *BAF* value, so it is in the interest of the facility to use the more complex expression. For implementation purposes though, it may be that in some cases supply chains are uniform enough that default values of *P* could be calculated, so that facilities would not need to perform these complex calculations.

The examples contained in this appendix show how to calculate *NBE* in a way that meticulously "follows the tons of biogenic CO_2" through a series of hypothetical stationary source production processes. In cases where the supply chain is long and complex, this quickly becomes a very complicated exercise. In application, it is unlikely that all of the relevant carbon masses will be measurable and known. As was described above for *L*, it may be necessary to estimate *P* without actually performing the full calculations described in these examples. Nonetheless, it is useful to think through this idealized application to better understand the *NBE* equation and how it is adaptable across different production processes and points of assessment.